Prelude: GR for the Common Man

Intro Cosmology Short Course

Lecture 1

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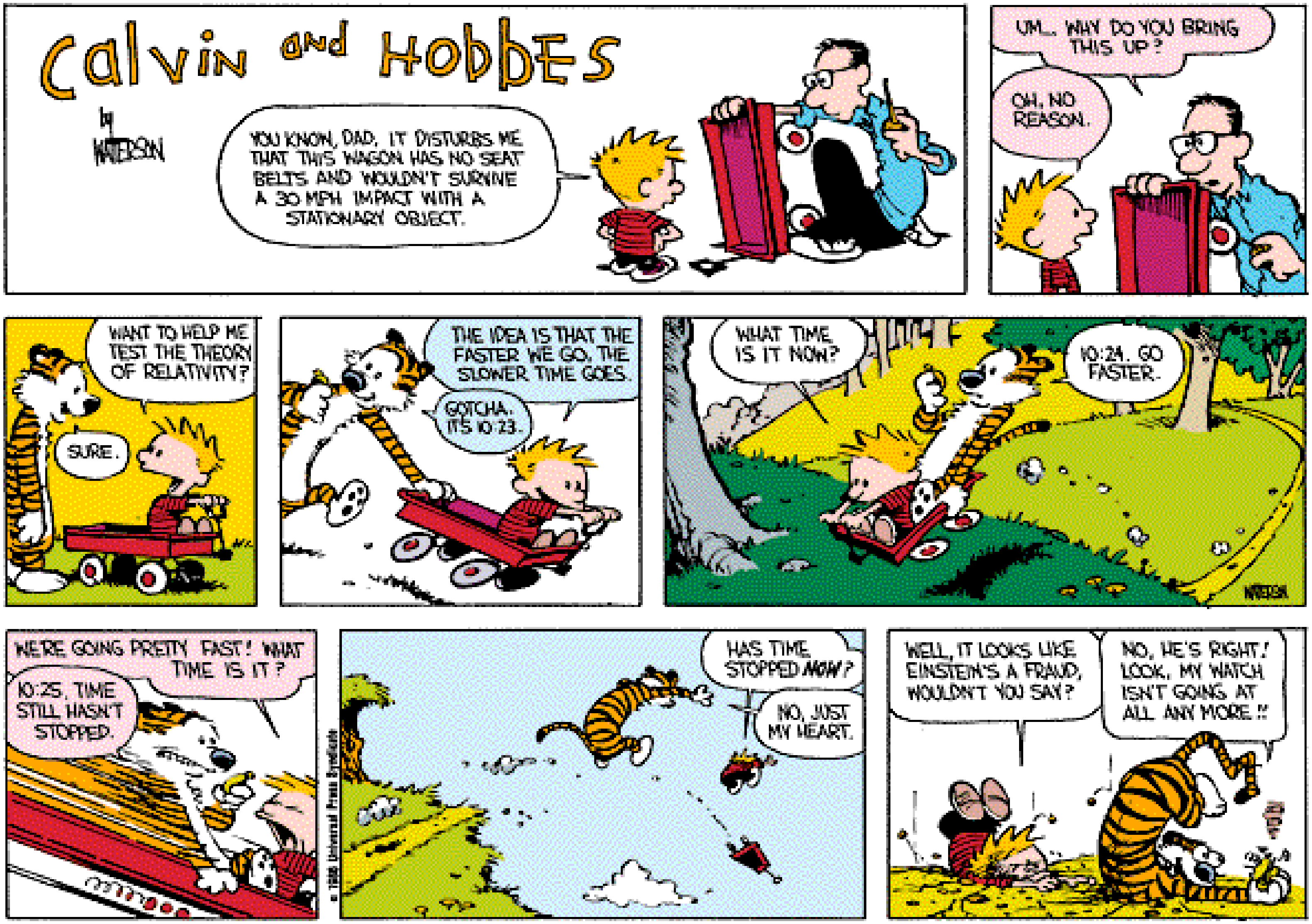
Calvin and HobbEs
by WATTERSON
YOU KNOW, DAD, IT DISTURBS ME THAT THIS WAGON HAS NO SEAT BELTS AND WOULDN'T SURVIVE A 30 MPH IMPACT WITH A STATIONARY OBJECT.
UM.. WHY DO YOU BRING THIS UP?
OH, NO REASON.
WANT TO HELP ME TEST THE THEORY OF RELATIVITY?
SURE.
THE IDEA IS THAT THE FASTER WE GO, THE SLOWER TIME GOES.
GOTCHA. IT'S 10:23.
WHAT TIME IS IT NOW?
10:24. GO FASTER.
WATTERSON
WE'RE GOING PRETTY FAST! WHAT TIME IS IT?
10:25. TIME STILL HASN'T STOPPED.
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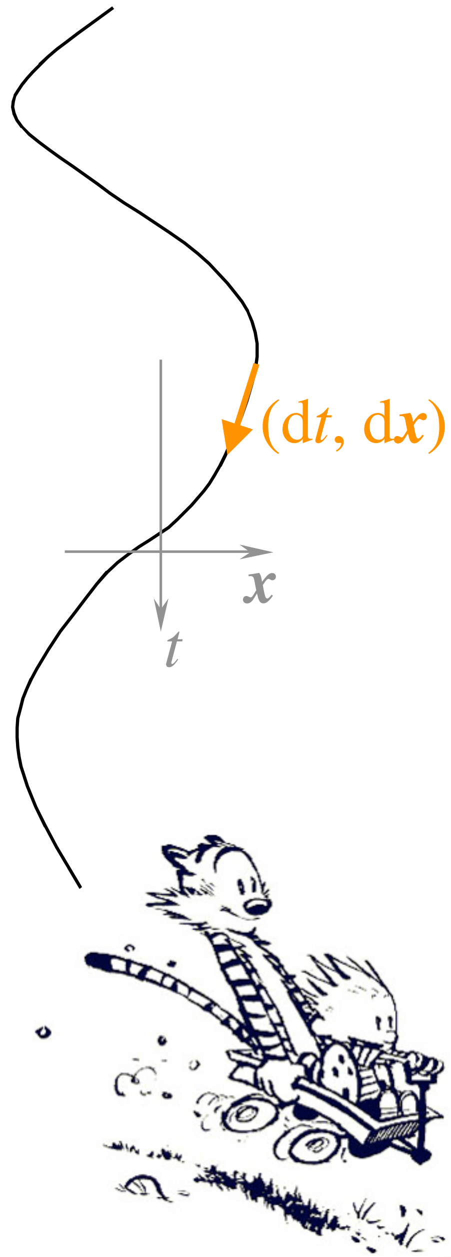
HAS TIME STOPPED NOW?
NO, JUST MY HEART.
WELL, IT LOOKS LIKE EINSTEIN'S A FRAUD, WOULDN'T YOU SAY?
NO, HE'S RIGHT! LOOK, MY WATCH ISN'T GOING AT ALL ANY MORE!!

What is Calvin & Hobbes' primary misconception?

Same path through space-time → Same subjective elapsed time

Subjective time -- "proper time" -- is a *fundamental* physical observable, independent of coordinates



Newtonian:

$$(dt, dx, dy, dz) \text{ at some point } (t, x, y, z)$$

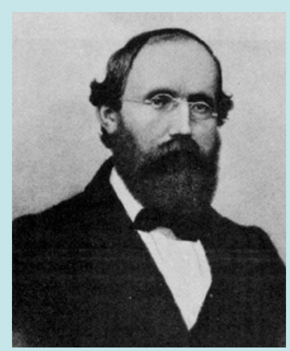
Most general:

$$(dx^0, dx^1, dx^2, dx^3) \text{ at } (x^0, x^1, x^2, x^3)$$

$$\underbrace{d\tau}_{\substack{\text{Proper}\\ \text{Time}}}{}^2 = \sum_{\mu,\nu=0}^{3} dx^\mu g_{\mu\nu}\left(x^0, x^1, x^2, x^3\right) dx^\nu$$

$g_{\mu\nu}$

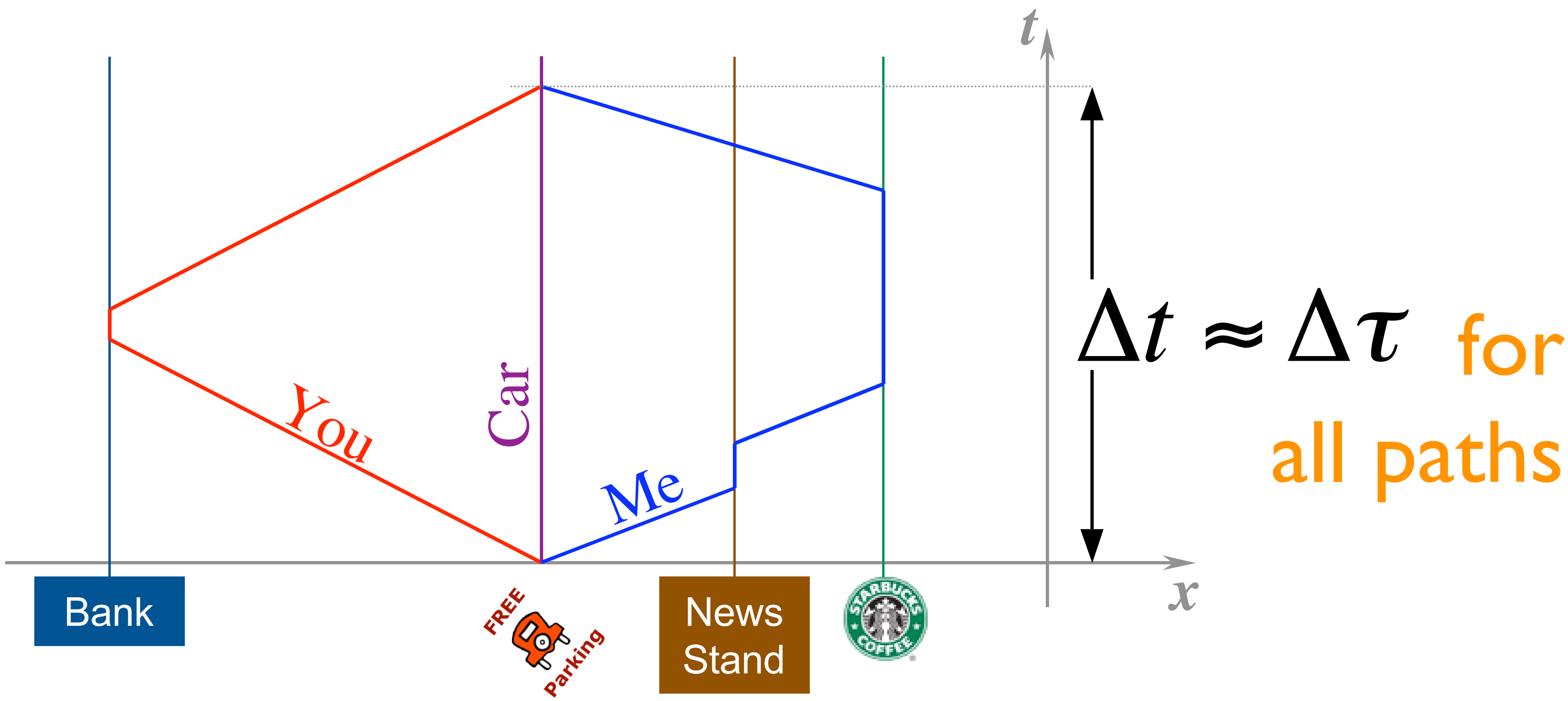
Metric Tensor



B. Riemann
German

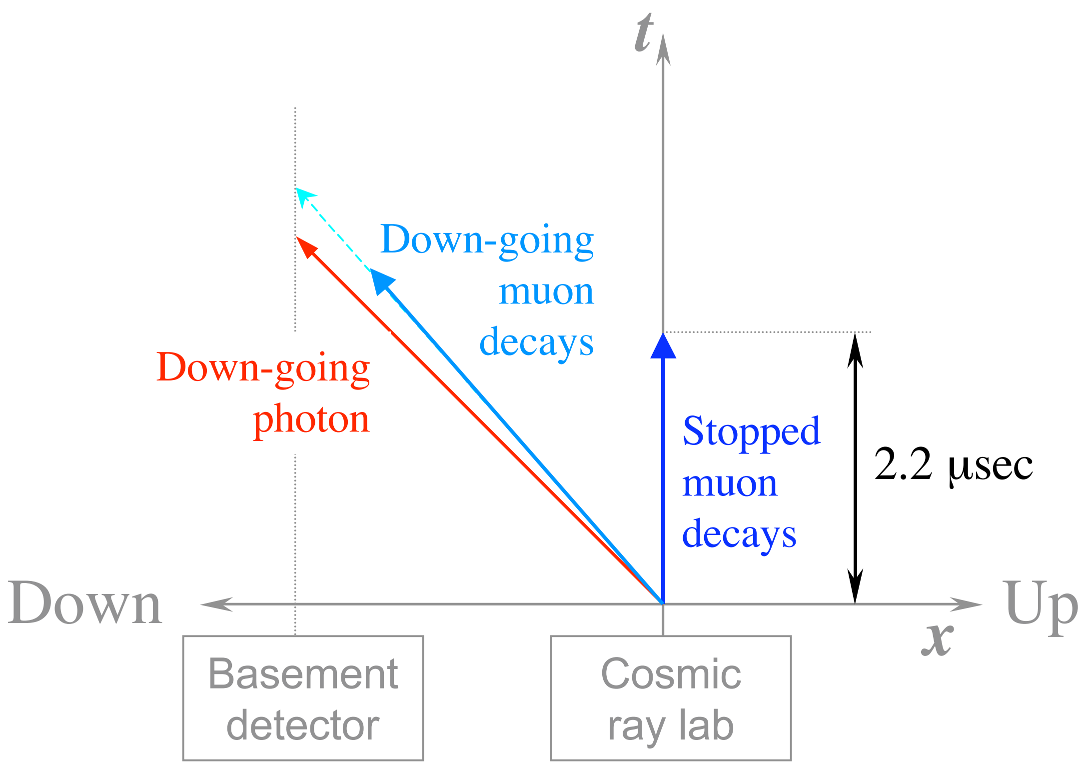
Formalized non-Euclidean geometry (1854)

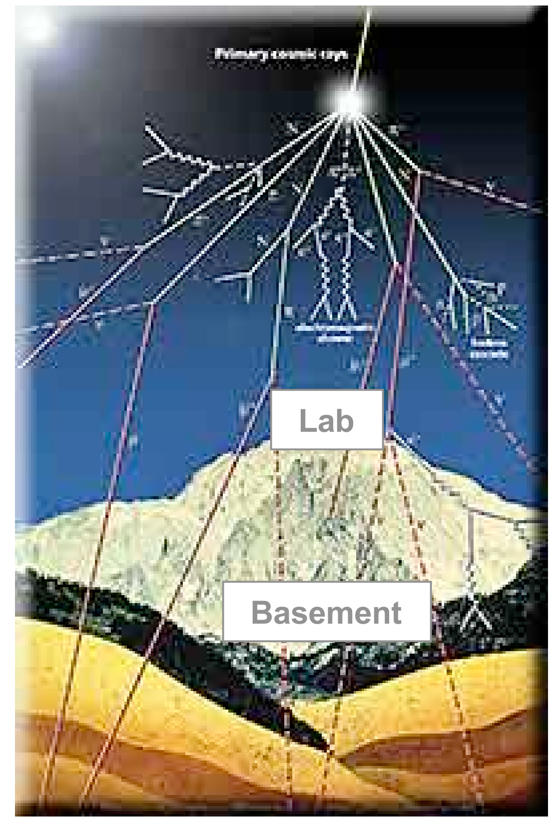
Assuming Newton found parking….



Galilean/
Newtonian

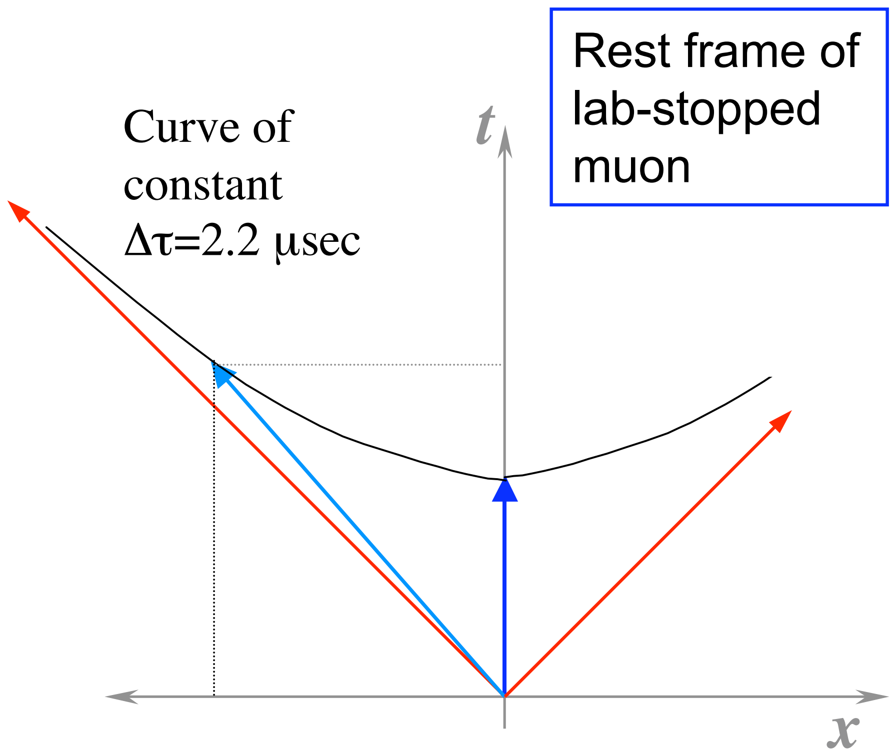
$$d\tau^2 \approx dt^2 \quad g_{00} \approx 1 \quad g_{0i}, g_{ij} \approx 0$$

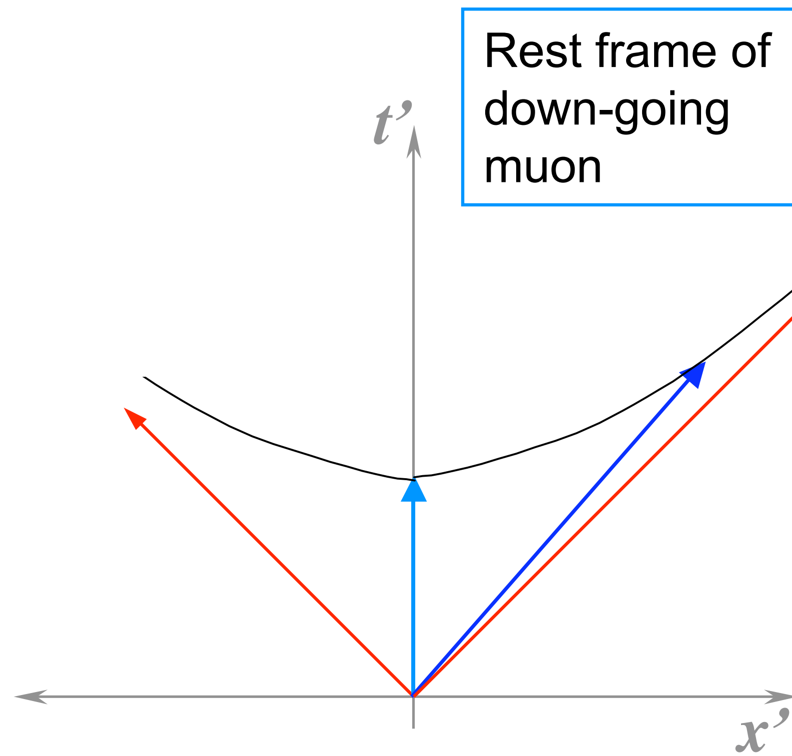




We observe:

1. Down-going muons $v < c$
2. Lifetime of down-going muons > 2.2μsec





$$d\tau^2 = dt^2 - dx^2/c^2$$

$$d\tau^2 = \left(dt'\right)^2 - \left(dx'\right)^2/c^2$$

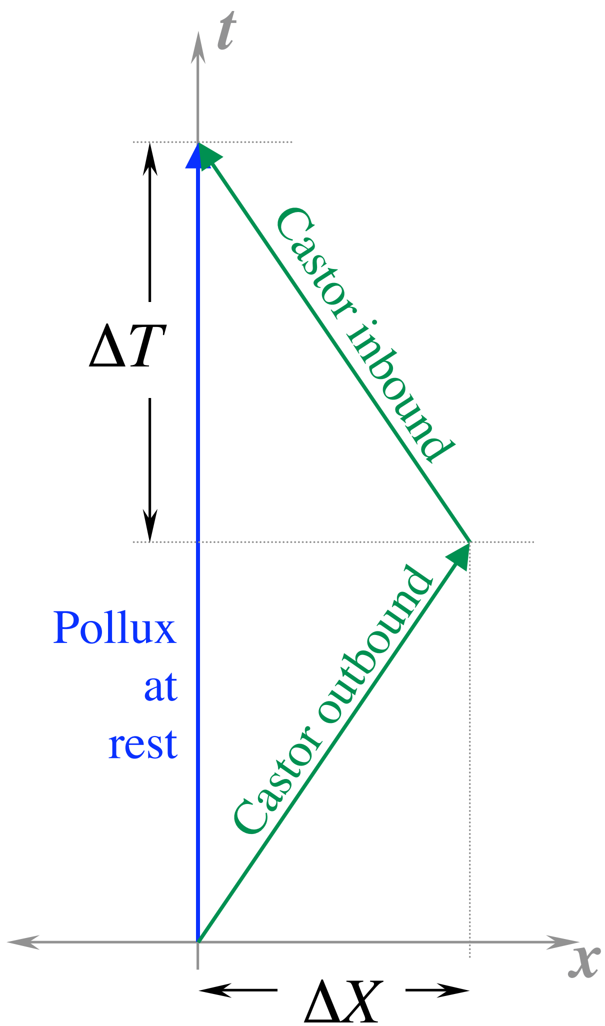
H. Minkowski German

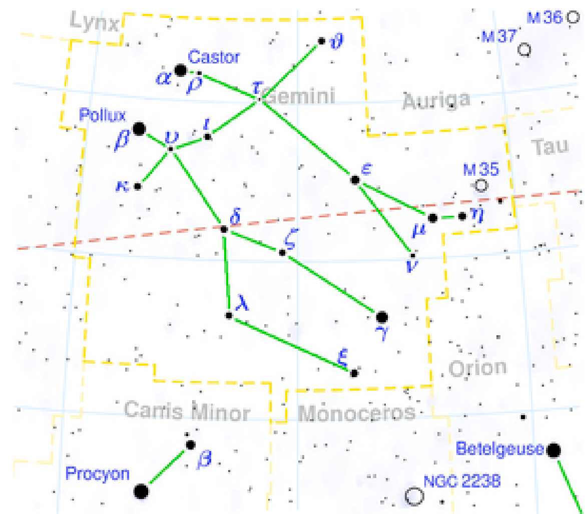
"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" (1907)

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/c^2 & 0 & 0 \\ 0 & 0 & -1/c^2 & 0 \\ 0 & 0 & 0 & -1/c^2 \end{bmatrix}$$

in *any* and *all* intertial frames

The Twin "Paradox" made easy





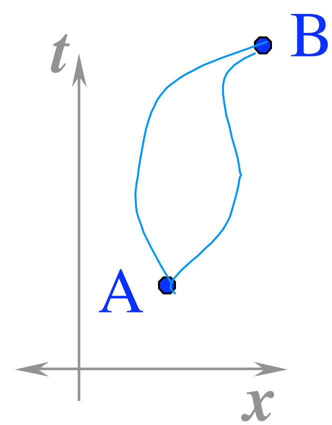
$$\tau_{\text{Pollux}} = 2\sqrt{\Delta T^2} = 2\Delta T$$

$$\tau_{\text{Castor}} = 2\sqrt{\Delta T^2 - \Delta X^2/c^2} < \tau_{\text{Pollux}}$$

It's just that simple!

New, generalized laws of motion

1. In getting from A to B, all free-falling objects will follow the path of **maximal proper time** (“geodesic”).



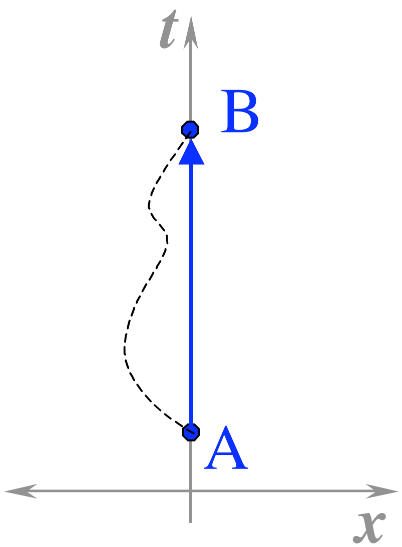
2. Photons follow “null” paths of **zero proper time**.

$$d\tau^2 = 0$$

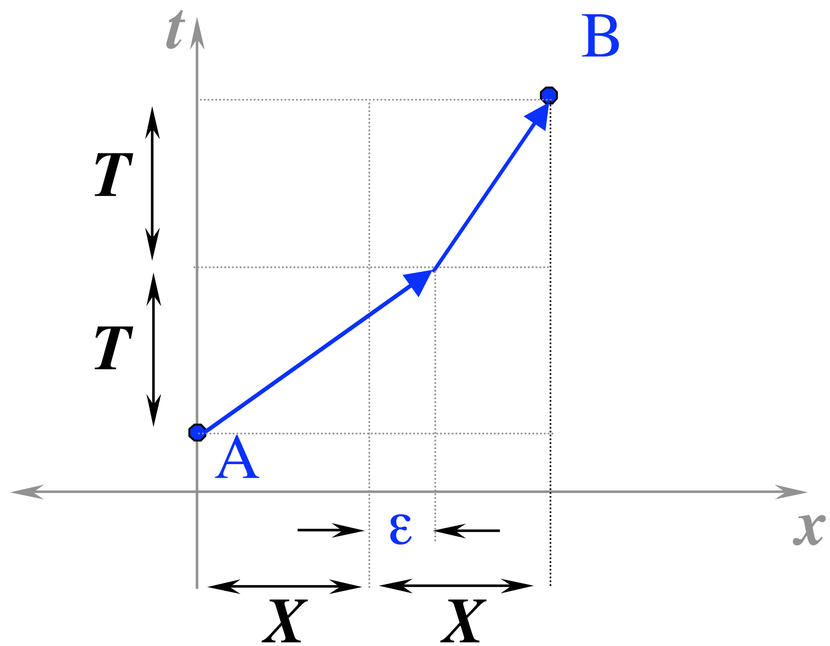
$$dt^2 - dx^2/c^2 = 0$$

$$\frac{dx}{dt} = \pm c$$

Recovering Newton's First Law



An object at rest tending to remain at rest

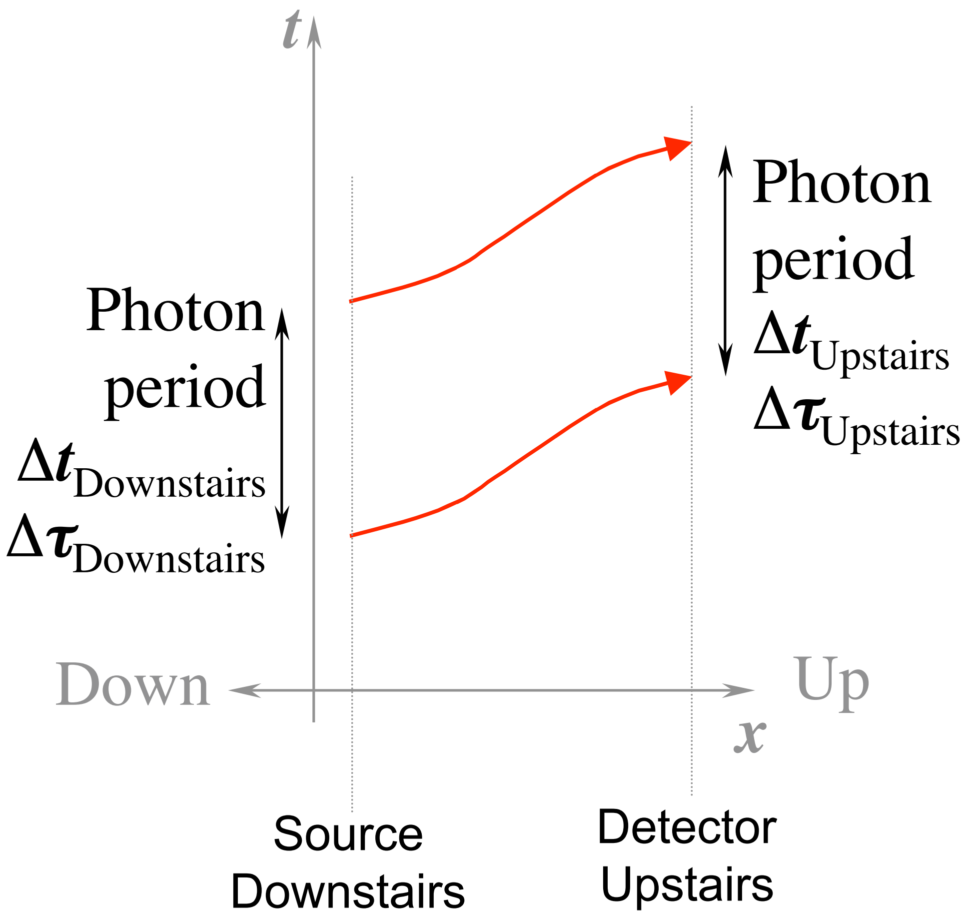


$$\Delta\tau_{AB}(\varepsilon) = \sqrt{T^2 - (X+\varepsilon)^2/c^2} + \sqrt{T^2 - (X-\varepsilon)^2/c^2}$$

Maximized with $\varepsilon = 0$, ie a straight line

$\Rightarrow$ Velocity remains constant

Gravitational Red Shift



$$\Delta t_{\text{Downstairs}} = \Delta t_{\text{Upstairs}}$$

but

$$\Delta \tau_{\text{Downstairs}} < \Delta \tau_{\text{Upstairs}}$$

so $\dfrac{d\tau}{dt} \neq$ constant over x

Let $\dfrac{d\tau}{dt} = 1 + \phi(x)/c^2$

Resulting metric:

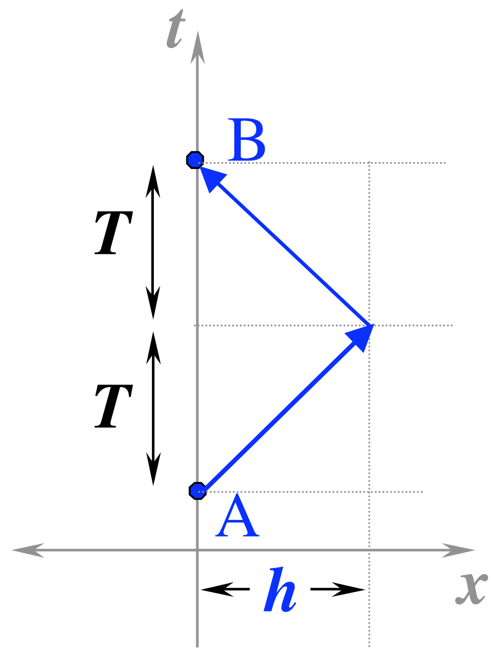
$$d\tau^2 = \left[1 + \phi(x)/c^2\right]^2 dt^2 - dx^2/c^2$$

Non-inertial frame -- curved space!

Motion in curved 1+1D space

$$d\tau^2 = \left[1 + \phi(x)/c^2\right]^2 dt^2 - dx^2/c^2$$

$$d\tau_{AB}(h) = 2\sqrt{[1+\phi(h/2)/c^2]^2 T^2 - h^2/c^2}$$



maximize $[1+\phi(h/2)/c^2]^2 T^2 - h^2/c^2$

$$\approx [1+2\phi(h/2)/c^2]T^2 - h^2/c^2$$

find $\boxed{h = (1/2)\,\phi'(h/2)T^2}$

so $$a = \frac{\Delta v}{T} = \frac{(-h/T)-(h/T)}{T} = -\phi'(h/2)$$

Conservative (Newtonian) Potential!

Force law: action at a distance

$$\vec{F}(\vec{x}) = \frac{GMm}{r^2}\hat{r}$$

$$\vec{a}(\vec{x}) = \frac{\vec{F}}{m} = \frac{GM}{r^2}\hat{r} = -\vec{\nabla}\left(-\frac{GM}{r}\right) = -\vec{\nabla}\,\phi(\vec{x})$$

Isaac Newton
British

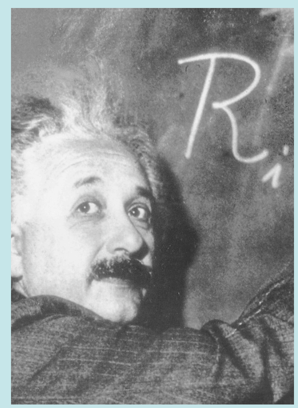
Universal Theory of Gravitation (1687)

Metric: a *local* property

$$d\tau^2 = \left[1 + \phi(\vec{x})/c^2\right]^2 dt^2 - d\vec{x}^2/c^2$$

$$+\,\text{maximum proper time principal}$$

$$\Rightarrow \vec{a}(\vec{x}) = -\vec{\nabla}\,\phi(\vec{x}) \quad \text{for} \quad \phi/c^2 << 1$$



Albert Einstein
German

General Theory of Relativity (1915)

Points to take home

- Subjective/proper time as the fundamental observable
- Central role of the metric
- Free-fall paths maximize subjective time
- Minkowski metric for empty space recovers Newton's 1st law
- Slightly curved space reproduces Newtonian gravitation